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Temperature characterization of dielectric-resonator materials

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Abstract

Dielectric resonators are widely used as components of microwave resonant cavities with extremely high Q factors. To design a temperature-compensated resonator it is necessary to know accurately the value of τ_ε, the temperature coefficient of the dielectric constant. A simple and accurate procedure is described to measure this quantity. Furthermore, the paper discusses uncertainties involved in the use of another quantity, τ_f, which is nowadays supplied by the manufacturers of dielectric resonators to characterize the temperature variation of dielectric resonators.

Keywords: Electromagnetic fields; Microwave dielectric properties; Microwave resonators; Temperature

1. Introduction

Microwave resonators are often built with the use of dielectric resonators to achieve a narrow-band, low-loss operation. For applications in wireless communication networks, the close spacing of individual channels requires a very stable resonant frequency, ideally invariant with the ambient temperature. The temperature stability of the resonant frequency is often required to be under one part per million, per degree centigrade (1 ppm/°C).

Two main mechanisms cause the resonant frequency to vary with temperature. First, dimensions L_i of the various parts constituting the resonator expand with temperature T. This is described by a linear temperature expansion coefficient α_i of the i-th part:

$$\alpha_i = \frac{1}{L_i}\frac{\Delta L_i}{\Delta T}\bigg|_{T=T_0} \tag{1}$$

As the resonator dimensions are proportional to the resonant wavelength, the resonant frequency will typically decrease with increasing dimensions.

Second, the dielectric constant ε_r (permittivity) of the dielectric resonator also changes with temperature T. The linear term of this variation is the temperature coefficient of the dielectric constant, τ_ε:

$$\tau_\varepsilon = \frac{1}{\varepsilon_r}\frac{\Delta \varepsilon_r}{\Delta T}\bigg|_{T=T_0} \tag{2}$$

Most dielectrics used to manufacture dielectric resonators exhibit a decrease in ε_r with an increase in temperature, so that τ_ε is a negative number. Since the resonant frequency is typically proportional to $\varepsilon_r^{-1/2}$, the decrease in ε_r causes an increase of resonant frequency with temperature. Thus, it is possible to design a composite resonant cavity, consisting of several dielectric and conductive parts, so that the frequency increments compensate each other and the overall frequency variation at room temperature T_0 becomes stationary. To achieve a resonator stability better than 1 ppm/°C, the designer must know the values of τ_ε's with an accuracy of 1–2%. Unfortunately, the typical data[1] on τ_ε are inaccurate by 8–25%.

The goal of this paper is to describe the measurement procedure which enables an accurate determination of τ_ε. Furthermore, the paper analyzes the common practice of measuring the frequency temperature coefficient τ_f, and points out inadequacies of this procedure.

2. Courtney method of τ_ε measurement

This measurement procedure consists of inserting a cylindrical sample of the dielectric material to be measured between two parallel conducting plates, as shown in Fig. 1. To determine the value of the dielectric constant ε_r of the material, it is necessary to measure the resonant frequency of the resonant mode TE_{011}, which typically occurs at microwave frequencies. Thus, to determine the value of τ_ε the entire device must be placed in a temperature chamber, and the resonant frequency must be measured as a function of temperature, as done

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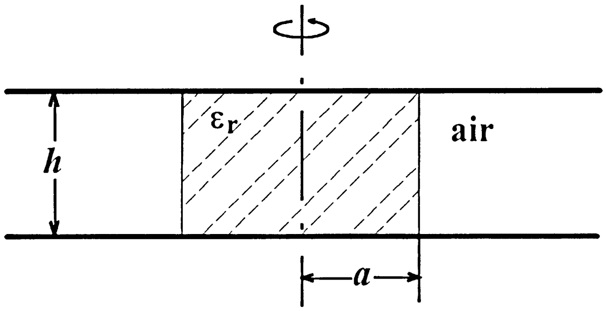

Fig. 1. Parallel-plate resonator used in Courtney's measurement.

by Courtney.[2] He has observed that, in a range of temperatures 0–100 °C, most of the dielectric materials exhibit a linear change of ε_r with temperature. The value of τ_ε is then obtained from the slope of this variation. Courtney's contribution was significant in that he has performed an analysis of systematic errors, and established the uncertainty limits of the procedure. His paper does not contain representative data on τ_ε of typical dielectric resonators, because these devices were not commercially available at that time. A more recent contribution to the subject of measuring τ_ε of dielectric resonators is by Mourachkine and Barel.[3]

From the historical perspective, the parallel plate configuration from Fig. 1 was first suggested by Hakki and Coleman.[4] The same configuration was analyzed by Cohn and Kelly[5] who have shown that the accuracy of the results is not affected by the presence of the small but unavoidable gaps between the resonator sample and the two ground planes. Kobayashi and Katoh[6] have analyzed how much the parallel plates should extend beyond the resonator diameter for a reliable measurement. As long as height h is smaller than one-half wavelength in free space, the radiation from the TE_{011} mode is insignificant, and the size of the parallel plates has little effect on the measured results. In addition to measuring the TE_{011} mode, several higher modes of the type TE_{0n1} (with $n=1$ to 4) can also be used[7] for the measurement of ε_r.

One difficulty in performing the measurement is the fact that the parallel-plate resonator in Fig. 1 supports many resonant modes so that before one starts taking the data he should be sure that the measured mode is TE_{011}. One simple check is to touch the edges of the open-ended resonator. For the unwanted modes, the display of the network analyzer will show a change in the depth of the resonant curve and also a shift in the resonant frequency, while modes TE_{0n1} will remain unaffected by such a small disturbance.

Once the resonant frequency f_0 of the TE_{0n1} mode has been found, the computation of ε_r can be done on a personal computer as follows. First, compute the radial wave number:

$$k_0 a = \frac{2\pi f_0 a}{c} \tag{3}$$

where c is the velocity of light in vacuum. Next, compute the argument y of the modified Bessel functions:

$$y = \sqrt{\left(\pi \frac{a}{h}\right)^2 - (k_0 a)^2} \tag{4}$$

Also, compute another auxiliary constant M:

$$M = \frac{K_1(y)}{y K_0(y)} \tag{5}$$

Then, find a zero of the transcendental function $\Phi(x)$:

$$\Phi(x) = J_1(x) + xMJ_0(x) = 0 \tag{6}$$

Function $\Phi(x)$ is finite and well behaved as shown in Fig. 2. Any primitive numerical procedure for solving linear equations should have no problems finding zeros of the function. The first zero corresponds to mode TE_{011}. Higher zeros, corresponding to modes TE_{0n1}, may be of interest when ε_r is to be measured in a wider frequency range with a single sample.[7] Once the value of x_n is found, the relative dielectric constant is computed from:

$$\varepsilon_r = \frac{x_n^2 + (\pi \frac{a}{h})^2}{(k_0 a)^2} \tag{7}$$

When the results of measurement in the temperature chamber are being processed, one has to take into account that dimensions a and h both change with temperature. For instance, the value of a at temperature T should be corrected to

$$a(T) = a(T_0)(1 + (T - T_0)\alpha) \tag{8}$$

and similarly for $h(T)$. The results of this computation provide the values of function $\varepsilon_r(T)$ and the value of τ_ε

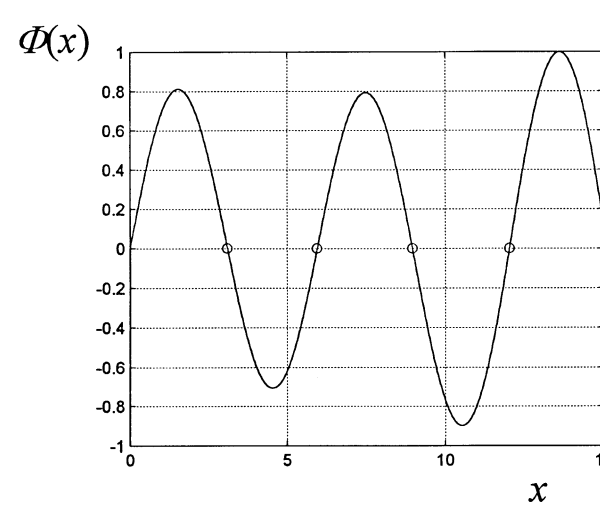

Fig. 2. First four zeros of function $\Phi(x)$.

is then evaluated from the slope of this function at T_0. A synthesized signal generator is preferred, because the frequency changes to be measured are extremely small. The program QZERO[8] was found to be useful in determining the exact value of the resonant frequency at each temperature.

3. Present practices

Although Courtney's measurement procedure is straightforward and very accurate, the major manufacturers of dielectric resonators have adopted a different experimental technique for characterizing the temperature variation of the dielectric material. They routinely specify the frequency temperature coefficient τ_f of the cylindrical conductor cavity which contains the dielectric resonator at its center. This coefficient is defined as follows:

$$\tau_f = \frac{1}{f}\frac{\Delta f}{\Delta T} \tag{9}$$

and is expressed in ppm/°C.

Fig. 3 shows the basic dimensions of such a test cavity. The cavity radius and height are approximately three times larger than the dielectric resonator's radius and height. A dielectric support keeps the dielectric resonator situated at the center of the cavity. The value of τ_f is measured by placing the entire cavity inside a temperature chamber that allows a controlled temperature variation. The resonant frequency of mode $TE_{01\delta}$ is then measured as a function of temperature and from the slope of that function one computes τ_f.

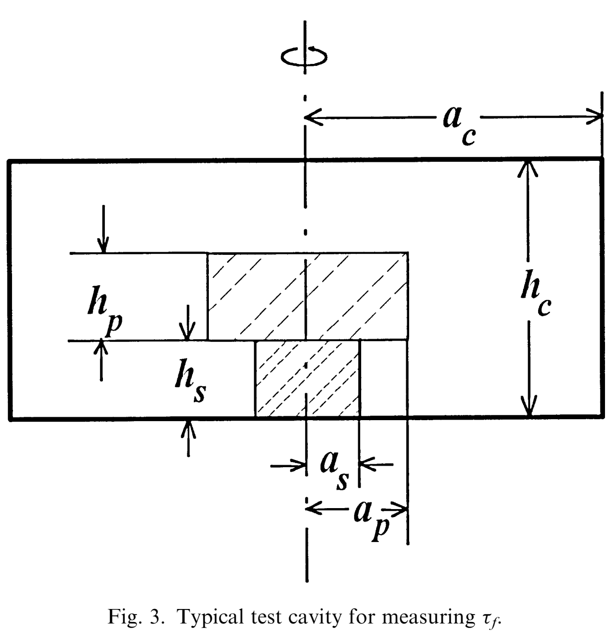

Fig. 3. Typical test cavity for measuring τ_f.

If the users of dielectric resonators would design their filters and oscillators by placing dielectric resonators inside the three-times larger conductor cavities, made of the same metal as the manufacturer's test cavities, then the value of τ_f would indeed accurately predict the temperature behavior of their devices. In practical life, miniaturization and low-cost requirements prohibit the use of such large cavities. Furthermore, many filters utilize a hybrid mode $HEM_{11\delta}$ instead of $TE_{01\delta}$ in order to create a dual resonance. In short, the value of τ_f provided by manufacturers is not applicable to any practical configurations.

The proper quantity to use for a systematic design of the temperature-compensated resonator is τ_ε and not τ_f. Manufacturer's publications and catalogs suggest that τ_ε can be computed from the knowledge of τ_f and α as follows:

$$\tau_\varepsilon = -2(\alpha + \tau_f) \tag{10}$$

The above equation holds exactly only for the resonant cavities homogeneously filled with the same dielectric material[1]. For composite resonators such as in Fig. 3, the variation of frequency with temperature is described by a linear model as follows:[9]

$$\tau_f = \sum_{i=1}^{3}(C_{ai} + C_{hi})\alpha_i + \sum_{j=1}^{2} C_{\varepsilon j}\tau_{\varepsilon j} \tag{11}$$

The above equation contains two summations: the first one describes the frequency changes due to the expansion of mechanical parts and the second one describes the frequency variation due to changes of dielectric constant with temperature. Index i identifies parts with various expansion coefficients α_i. The resonator shown in Fig. 3, consists of three parts: dielectric resonator ($i=1$ or p for "puck"), dielectric support ($i=2$ or s for "support") and conductor cavity ($i=3$ or c for "conductor").

Coefficients C_{ai} and C_{hi} are the sensitivity coefficients of the resonant frequency with respect to dimensions a_i and h_i:

$$C_{ai} = \frac{a_i}{f}\frac{\Delta f}{\Delta a_i}; \quad C_{hi} = \frac{h_i}{f}\frac{\Delta f}{\Delta h_i} \tag{12}$$

Index j in the second summation identifies the dielectric parts with various dielectric temperature coefficients: puck ($j=1$ or$p$) and support ($j=2$ or s). The third dielectric material in the cavity is air, but it is assumed that the dielectric constant of air is unity regardless of temperature, so its τ_ε is zero. Coefficients $C_{\varepsilon j}$ in the second summation are the sensitivities of the resonant frequency with respect to changes of the dielectric constant:

$$C_{\varepsilon j} = \frac{\varepsilon_{rj}}{f}\frac{\Delta f}{\Delta \varepsilon_{rj}} \tag{13}$$

The role that individual resonator parts play in the overall frequency variation with temperature can be best comprehended with a practical example. The values are selected to correspond to Murata resonator DRD293UA130[10] intended for operation from 1.82 to 1.97 GHz. It is assumed that all the dimensions and material properties are known, except for the dielectric temperature coefficient τ_ε. Using (11), the desired coefficient will be computed as follows:

$$\tau_{\varepsilon p} = \frac{1}{C_{\varepsilon p}}\left[-C_{\varepsilon s}\tau_{\varepsilon s} - \sum_{i=1}^{3}(C_{ai} + C_{hi})\alpha_i + \tau_f\right] \quad (14)$$

The dielectric resonator dimensions and properties are taken to be:

$\varepsilon_{rp} = 36.6$, $\tau_f = -4$ ppm/°C, $\alpha_p = 6$ to 7 ppm/°C, $a_p = 14.67$ mm, $h_p = 13.02$ mm.

The conductor cavity is assumed to be made of aluminum as follows:

$\alpha_c = 23.12$ ppm/°C, $a_c = 44.02$ mm, $h_c = 39.06$ mm.

The support is not a solid cylinder as in Fig. 3, but is made of alumina ceramics Al_2O_3 of a tubular shape (inner and outer radii are a_{is} and a_{os}):

$a_{is} = 3.87$ mm, $a_{os} = 7.91$ mm, $h_s = 11.76$ mm, $\alpha_s = 6.4$, $\varepsilon_r = 9.7$, $\tau_{\varepsilon s} = 116$ ppm/°C.

Coefficients C_{ai} and C_{hi} can be evaluated with the use of a numerical procedure for computation of the electromagnetic field of mode $TE_{o1\delta}$ inside the test cavity. A relatively simple program was used for this purpose, as the only goal of this exercise is to find the orders of magnitudes involved, and not to actually design a compensated resonator. The program is intended to analyze a dielectric resonator which is placed on a solid dielectric slab covering the bottom of a cylindrical cavity. The configuration is shown in Fig. 4. Thus, to apply the program to the test cavity configuration, the tubular dielectric support has to be replaced by an equivalent dielectric slab with an effective dielectric constant lower than ε_{rs}. This is done by taking the effective dielectric constant to be proportional to the ratio of volumes:

$$\varepsilon_{r\text{seff}} = 1 + (\varepsilon_{rs} - 1)\frac{V_s}{V_a} \quad (15)$$

V_s is the volume of the tubular spacer:

$$V_s = \left(a_{os}^2 - a_{is}^2\right)\pi h_s \quad (16)$$

and V_a is the volume of the active space below the puck:

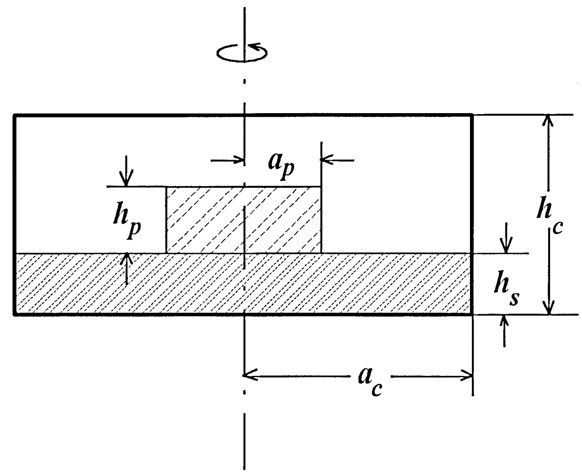

Fig. 4. Simplified model of the test cavity, used in computation of sensitivity coefficients.

$$V_a = a_p^2\pi h_s \quad (17)$$

For the dimensions given above, the effective relative dielectric constant of the support comes out to be $\varepsilon_{r\text{seff}} = 1.92$. The results of numerically evaluating the sensitivity coefficients by finite differences are shown in Table 1. Since the tubular support is now replaced by an equivalent dielectric slab filling the bottom part of the entire cavity, the coefficient C_{as} becomes meaningless, so its value is marked "ignored." The resonant frequency evaluated by the program is 1.809 GHz.

When these values are substituted in (14), the desired value is found to be $\tau_{\varepsilon p} = -5.9328$. On the other hand, the direct use of (10) yields $\tau_{\varepsilon p} = -2(6.5–4) = -5.0$, a number that differs by 16%. Such an uncertainty is unacceptable for design purposes.

The distribution of individual contributions is displayed in Table 2. The smallest contribution to the measured value of $\tau_{\varepsilon p}$ comes from the expansion of the support $\tau_{\varepsilon s}$. The reason is that the dielectric resonator is centered in the cavity, thus the resonant frequency is insensitive to incremental moves from this neutral position. All the other contributions are larger than 17% of the final result. The largest two contributions come from α_p and from τ_f as the approximate Eq. (10) suggests. But the other contributions, except the contribution from α_s, are not negligible at all.

From the above discussion, it becomes obvious that using Eq. (10) and the value of τ_f supplied by the manufacturer yields a totally unreliable value of τ_ε. The only

Table 1
Sensitivity coefficients

	Puck	Support	Conductor
C_a	−0.6730	Ignored	0.0134
C_h	−0.2628	−0.0112	0.0397
C_ε	−0.4894	−0.0041	N/A

Table 2
Individual contributions to $\tau_{\varepsilon p}$

	Accurate Ppm/°C	Approximate ppm/°C
From $\tau_{\varepsilon s}$	+0.9776	0
From α_p	−12.4261	−13.0
From α_s	−0.1463	0
From α_c	−2.5100	0
From τ_f	+8.1720	+8.0
Total	−5.9328	−5.0

legitimate use of τ_f is for comparing the temperature properties of two dielectric resonators made of different materials. The one which has a more positive value of τ_f is the one that has a steeper negative slope τ_ε.

4. Conclusions

When the material properties of dielectric resonators are known with sufficient precision, it is possible to design temperature-compensated composite resonant cavities with the temperature stability better than 1 ppm/°C. The critical material property is τ_ε, the temperature coefficient of the dielectric constant. At present, manufacturers of dielectric resonators do not measure this quantity. Instead, they characterize the temperature behavior of their materials with a different quantity τ_f, the temperature coefficient of the resonant frequency of a test cavity.

This paper demonstrates that τ_f is of little help in designing temperature-stable microwave resonators that contain dielectric resonators. The paper also reminds the readers that a straightforward and accurate procedure for measurement of τ_ε is available. It is hoped that, at least for higher-priced premium-quality materials, some manufacturer will begin performing the measurement and providing the users with reliable values of τ_ε.

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